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stack.py

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ICPC Regional Jakarta 2022

Nov 27, 2022

Contest (1)

template.cpp

15 lines

```cpp
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
#define trav(x, v) for(auto &x : v)
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

int main() {
  cin.tie(0)->sync_with_stdio(0);
  cin.exceptions(cin.failbit);
}
```

template.py

Description: Fast I/O. Improves performance when reading many ($\geq 10^4$) lines. Returns an empty string instead of raising EOFError!

5 lines

```python
import sys
sys.setrecursionlimit(10**9)
input = lambda: sys.stdin.readline().rstrip('\r\n')
# Many calls to print is also slow, consider:
print('\n'.join(buffered_output))
```

.bashrc

12 lines

```
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps = <>
go() {
  # -Wall -Wshadow -Wreturn-type -Wunused nyalain aja! pasti
      berguna
  # glibxx debug buat ngecek out of bound di vector
  # -fsanitize=undefined,address buat ngecek out of bound di
      array
  # -Wl,--stack,1073741824 buat stack size
  g++ -std=c++17 -Wall -Wshadow -Wreturn-type -Wunused -
      D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC -fsanitize=
      undefined,address "$1".cpp -o $1
}

gg() {
  ./$1 < in
}
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed - bf}{ad - bc} \\ y &= \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A'_i}{\det A}$$

where A'_i is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k + c_1 x^{k-1} + \cdots + c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.3 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v + w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc\left[1 - \left(\frac{a}{b + c}\right)^2\right]}$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\frac{a + b}{a - b} = \frac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}}$

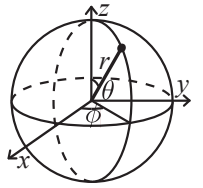
2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$.

2.4.3 Spherical coordinates



$$\begin{aligned} x &= r \sin \theta \cos \phi \\ y &= r \sin \theta \sin \phi \\ z &= r \cos \theta \end{aligned} \qquad \begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} \\ \theta &= \text{acos}(z/\sqrt{x^2 + y^2 + z^2}) \\ \phi &= \text{atan2}(y, x) \end{aligned}$$

2.5 Derivatives/Integrals

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx} \tan x = 1 + \tan^2 x \qquad \frac{d}{dx} \arctan x = \frac{1}{1 + x^2}$$
$$\int \tan ax = -\frac{\ln |\cos ax|}{a} \qquad \int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \text{erf}(x) \qquad \int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$$

Integration by parts:

$$\int_a^b f(x) g(x) dx = [F(x) g(x)]_a^b - \int_a^b F(x) g'(x) dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$ $\quad$ $\sum_{k=0}^{n} k^2\binom{n}{k} = (n+n^2)2^{n-2}$

$sum_{j=0}^{k}\binom{m}{j}\binom{n-m}{k-j} = \binom{n}{k}$ $\quad$ $\sum_{m=0}^{n}\binom{m}{j}\binom{n-m}{k-j} = \binom{n+1}{k+1}$

$\sum_{m=0}^{n}\binom{m}{k} = \binom{n+1}{k+1}$ $\quad$ $\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor}\binom{n-k}{k} = F(n+1)$

$\sum_{j=0}^{m}\binom{m}{j}^2 = \binom{2m}{m}$ $\quad$ $\sum_{i=0}^{n} i\binom{n}{i}^2 = \frac{n}{2}\binom{2n}{n}$

$\sum_{i=0}^{n} i^2\binom{n}{i}^2 = n^2\binom{2n-2}{n-1}$ $\quad$ $\sum_{k=q}^{n}\binom{n}{k}\binom{k}{q} = 2^{n-q}\binom{n}{q}$

$\sum_{k=-a}^{a}(-1)^k\binom{2a}{k+a}^3 = \frac{(3a)!}{a!^3}$

$$\sum_{k=-a}^{a}(-1)^k\binom{a+b}{a+k}\binom{b+c}{b+k}\binom{c+a}{c+k} = \frac{(a+b+c)!}{a!b!c!}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n, p)$, $n = 1, 2, \ldots, 0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n, p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\mathrm{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

Data structures (3)

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
Time: $\mathcal{O}(\log N)$

782797, 16 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}
```

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

d77092, 7 lines

```
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
  const uint64_t C = ll(4e18 * acos(0)) | 71;
  ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};
__gnu_pbds::gp_hash_table<ll,int,chash> h({},{},{},{},{1<<16});
```

WaveletTree.h

Description: Tree that recursively partitions a sequence into subsequences by value. σ is the difference between the largest and smallest element in the sequence (pre-compress values if σ is large). Use with a bump allocator for better performance.
Usage: `auto [lo, hi] = minmax_element(all(A));`
`Node tr(A, *lo, (*hi)+1);`
Time: $\mathcal{O}(\log \sigma)$. **Space:** $\mathcal{O}(N \log \sigma)$.
71870e, 35 lines

```
struct Node {
  Node *l = 0, *r = 0;
  int lo, hi; vi C; // C[i] = # of first i elements going left
  Node(const vi& A, int lo, int hi) : lo(lo), hi(hi), C(1, 0) {
    if(lo + 1 == hi) return;
    int mid = (lo + hi) / 2;
    vi L, R;
    for(int a: A) {
      C.push_back(C.back());
      if(a < mid) L.push_back(a), C.back()++;
      else R.push_back(a);
    }
    l = new Node(L, lo, mid), r = new Node(R, mid, hi);
  }
  // k'th (0-indexed) element in the sorted range [L, R)
  int quantile(int k, int L, int R) {
    if(lo + 1 == hi) return lo;
    int c = C[R] - C[L];
    if(k < c) return l->quantile(k, C[L], C[R]);
    return r->quantile(k - c, L - C[L], R - C[R]);
  }
  // number of elements in range [0, R) equal to x
  int rank(int x, int R) {
    if(lo + 1 == hi) return R;
    if(x < l->hi) return l->rank(x, C[R]);
    return r->rank(x, R - C[R]);
  }
  // number of elements x in range [L, R) st. a <= x < b
  int rectangle(int a, int b, int L, int R) {
    if(a <= lo && hi <= b) return R - L;
    if(a >= hi || b <= lo) return 0;
    return l->rectangle(a, b, C[L], C[R]) +
      r->rectangle(a, b, L - C[L], R - C[R]);
  }
};
```

LiChaoTree.h

Description: Jago
06b342, 90 lines

```
template<typename data_t>
struct Line {
  data_t a, b;
  Line() : a(0), b(-inf) {}
  Line(data_t a, data_t b) : a(a), b(b) {}
  data_t get(data_t x) {
    return a * x + b;
  }
  void add(Line x) {
    a += x.a;
    b += x.b;
  }
};
struct Node {
  Line<data_t> line = Line<data_t>();
  Line<data_t> lazy = Line<data_t>(0, 0);
  Node *lc = nullptr, *rc = nullptr;
  void apply(data_t l, data_t r, Line<data_t> v) {
    line.add(v); lazy.add(v);
  }
};
void PushLazy(Node* &n, data_t tl, data_t tr) {
  if (n == nullptr) return;
  if (n->lc == nullptr) n->lc = new Node();
  if (n->rc == nullptr) n->rc = new Node();
  data_t mid = (tl + tr) / 2;
  n->lc->apply(tl, mid, n->lazy);
  n->rc->apply(mid + 1, tr, n->lazy);
  n->lazy = Line<data_t>(0, 0);
}
void PushLine(Node* &n, data_t tl, data_t tr) {
  if (n == nullptr) return;
  data_t mid = (tl + tr) / 2;
  InsertLineKnowingly(n->lc, tl, mid, n->line);
  InsertLineKnowingly(n->rc, mid + 1, tr, n->line);
  n->line = Line<data_t>();
}
void InsertLineKnowingly(Node* &n, data_t tl, data_t tr, Line<
    data_t> x) {
  if (n == nullptr) n = new Node();
  if (n->line.get(tl) < x.get(tl)) swap(n->line, x);
  if (n->line.get(tr) >= x.get(tr)) return;
  if (tl == tr) return;
  data_t mid = (tl + tr) / 2;
  PushLazy(n, tl, tr);
  if (n->line.get(mid) > x.get(mid)) {
    InsertLineKnowingly(n->rc, mid + 1, tr, x);
  } else {
    swap(n->line, x);
    InsertLineKnowingly(n->lc, tl, mid, x);
  }
}
void InsertLine(Node* &n, data_t tl, data_t tr, data_t l,
    data_t r, Line<data_t> x) {
  if (tr < l || r < tl || tl > tr || l > r) return;
  if (n == nullptr) n = new Node();
  if (l <= tl && tr <= r) return InsertLineKnowingly(n, tl, tr,
      x);
  data_t mid = (tl + tr) / 2;
  PushLazy(n, tl, tr);
  InsertLine(n->lc, tl, mid, l, r, x);
  InsertLine(n->rc, mid + 1, tr, l, r, x);
}
void AddLine(Node* &n, data_t tl, data_t tr, data_t l, data_t r
    , Line<data_t> x) {
  if (tr < l || r < tl || tl > tr || l > r) return;
  if (n == nullptr) n = new Node();
  if (l <= tl && tr <= r) return n->apply(tl, tr, x);
  data_t mid = (tl + tr) / 2;
  PushLazy(n, tl, tr); PushLine(n, tl, tr);
  AddLine(n->lc, tl, mid, l, r, x);
  AddLine(n->rc, mid + 1, tr, l, r, x);
}
data_t Query(Node* &n, data_t tl, data_t tr, data_t x) {
  if (n == nullptr) return -inf;
  if (tl == tr) return n->line.get(x);
  data_t res = n->line.get(x);
  data_t mid = (tl + tr) / 2;
  PushLazy(n, tl, tr);
  if (x <= mid)
    res = max(res, Query(n->lc, tl, mid, x));
  else
    res = max(res, Query(n->rc, mid + 1, tr, x));
  return res;
}
void InsertLine(data_t l, data_t r, Line<data_t> x) {
  return InsertLine(root, 0, sz - 1, l, r, x);
}
void AddLine(data_t l, data_t r, Line<data_t> x) {
  return AddLine(root, 0, sz - 1, l, r, x);
}
data_t Query(data_t x) {
  return Query(root, 0, sz - 1, x);
}
```

UnionFind.h

Description: Disjoint-set data structure.
Time: $\mathcal{O}(\alpha(N))$
7aa27c, 14 lines

```
struct UF {
  vi e;
  UF(int n) : e(n, -1) {}
  bool sameSet(int a, int b) { return find(a) == find(b); }
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : e[x] = find(e[x]); }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    e[a] += e[b]; e[b] = a;
    return true;
  }
};
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
Usage: `int t = uf.time(); ...; uf.rollback(t);`
Time: $\mathcal{O}(\log(N))$
de4ad0, 21 lines

```
struct RollbackUF {
  vi e; vector<pii> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return sz(st); }
  void rollback(int t) {
    for (int i = time(); i --> t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a]});
    st.push_back({b, e[b]});
    e[a] += e[b]; e[b] = a;
    return true;
  }
};
```

SubMatrix.h

Description: Calculate submatrix sums quickly, given upper-left and lower-right corners (half-open).
Usage: `SubMatrix<int> m(matrix);`
`m.sum(0, 0, 2, 2); // top left 4 elements`
Time: $\mathcal{O}\left(N^2 + Q\right)$
c59ada, 13 lines

```
template<class T>
struct SubMatrix {
  vector<vector<T>> p;
  SubMatrix(vector<vector<T>>& v) {
    int R = sz(v), C = sz(v[0]);
    p.assign(R+1, vector<T>(C+1));
    rep(r,0,R) rep(c,0,C)
      p[r+1][c+1] = v[r][c] + p[r][c+1] + p[r+1][c] - p[r][c];
  }
  T sum(int u, int l, int d, int r) {
    return p[d][r] - p[d][l] - p[u][r] + p[u][l];
```

```
  }
};
```

Matrix.h

Description: Basic operations on square matrices.
Usage: `Matrix<int, 3> A;`
`A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};`
`vector<int> vec = {1,2,3};`
`vec = (A^N) * vec;`

c43c7d, 26 lines

```
template<class T, int N> struct Matrix {
  typedef Matrix M;
  array<array<T, N>, N> d{};
  M operator*(const M& m) const {
    M a;
    rep(i,0,N) rep(j,0,N)
      rep(k,0,N) a.d[i][j] += d[i][k]*m.d[k][j];
    return a;
  }
  vector<T> operator*(const vector<T>& vec) const {
    vector<T> ret(N);
    rep(i,0,N) rep(j,0,N) ret[i] += d[i][j] * vec[j];
    return ret;
  }
  M operator^(ll p) const {
    assert(p >= 0);
    M a, b(*this);
    rep(i,0,N) a.d[i][i] = 1;
    while (p) {
      if (p&1) a = a*b;
      b = b*b;
      p >>= 1;
    }
    return a;
  }
};
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

8ec1c7, 30 lines

```
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

Treap.h

Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: $\mathcal{O}(\log N)$

9556fc, 55 lines

```
struct Node {
  Node *l = 0, *r = 0;
  int val, y, c = 1;
  Node(int val) : val(val), y(rand()) {}
  void recalc();
};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {
  if (n) { each(n->l, f); f(n->val); each(n->r, f); }
}

pair<Node*, Node*> split(Node* n, int k) {
  if (!n) return {};
  if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(k)
    auto pa = split(n->l, k);
    n->l = pa.second;
    n->recalc();
    return {pa.first, n};
  } else {
    auto pa = split(n->r, k - cnt(n->l) - 1); // and just "k"
    n->r = pa.first;
    n->recalc();
    return {n, pa.second};
  }
}

Node* merge(Node* l, Node* r) {
  if (!l) return r;
  if (!r) return l;
  if (l->y > r->y) {
    l->r = merge(l->r, r);
    l->recalc();
    return l;
  } else {
    r->l = merge(l, r->l);
    r->recalc();
    return r;
  }
}

Node* ins(Node* t, Node* n, int pos) {
  auto pa = split(t, pos);
  return merge(merge(pa.first, n), pa.second);
}

// Example application: move the range [l, r) to index k
void move(Node*& t, int l, int r, int k) {
  Node *a, *b, *c;
  tie(a,b) = split(t, l); tie(b,c) = split(b, r - l);
  if (k <= l) t = merge(ins(a, b, k), c);
  else t = merge(a, ins(c, b, k - r));
}
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
Time: Both operations are $\mathcal{O}(\log N)$.

e62fac, 22 lines

```
struct FT {
  vector<ll> s;
  FT(int n) : s(n) {}
  void update(int pos, ll dif) { // a[pos] += dif
    for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
  }
  ll query(int pos) { // sum of values in [0, pos)
    ll res = 0;
    for (; pos > 0; pos &= pos - 1) res += s[pos-1];
    return res;
  }
  int lower_bound(ll sum) {// min pos st sum of [0, pos] >= sum
    // Returns n if no sum is >= sum, or -1 if empty sum is.
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) {
      if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
        pos += pw, sum -= s[pos-1];
    }
    return pos;
  }
};
```

FenwickTree2d.h

Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).
Time: $\mathcal{O}\left(\log^2 N\right)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h" 157f07, 22 lines

```
struct FT2 {
  vector<vi> ys; vector<FT> ft;
  FT2(int limx) : ys(limx) {}
  void fakeUpdate(int x, int y) {
    for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
  }
  void init() {
    for (vi& v : ys) sort(all(v)), ft.emplace_back(sz(v));
  }
  int ind(int x, int y) {
    return (int)(lower_bound(all(ys[x]), y) - ys[x].begin()); }
  void update(int x, int y, ll dif) {
    for (; x < sz(ys); x |= x + 1)
      ft[x].update(ind(x, y), dif);
  }
  ll query(int x, int y) {
    ll sum = 0;
    for (; x; x &= x - 1)
      sum += ft[x-1].query(ind(x-1, y));
    return sum;
  }
};
```

RMQ.h

Description: Range Minimum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time.
Usage: `RMQ rmq(values);`
`rmq.query(inclusive, exclusive);`
Time: $\mathcal{O}(|V| \log |V| + Q)$

510c32, 16 lines

```
template<class T>
struct RMQ {
  vector<vector<T>> jmp;
  RMQ(const vector<T>& V) : jmp(1, V) {
```

```
    for (int pw = 1, k = 1; pw * 2 <= sz(V); pw *= 2, ++k) {
      jmp.emplace_back(sz(V) - pw * 2 + 1);
      rep(j,0,sz(jmp[k]))
        jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
    }
  }
  T query(int a, int b) {
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
  }
};
```

MoQueries.h

Description: Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change `step` to add/remove the edge (a, c) and remove the initial `add` call (but keep `in`).
Time: $\mathcal{O}\left(N\sqrt{Q}\right)$

a12ef4, 49 lines

```
void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer

vi mo(vector<pii> Q) {
  int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s;
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) {
    pii q = Q[qi];
    while (L > q.first) add(--L, 0);
    while (R < q.second) add(R++, 1);
    while (L < q.first) del(L++, 0);
    while (R > q.second) del(--R, 1);
    res[qi] = calc();
  }
  return res;
}

vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root=0){
  int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
  add(0, 0), in[0] = 1;
  auto dfs = [&](int x, int p, int dep, auto& f) -> void {
    par[x] = p;
    L[x] = N;
    if (dep) I[x] = N++;
    for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
    if (!dep) I[x] = N++;
    R[x] = N;
  };
  dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) rep(end,0,2) {
    int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                  else { add(c, end); in[c] = 1; } a = c; }
    while (!(L[b] <= L[a] && R[a] <= R[b]))
      I[i++] = b, b = par[b];
    while (a != b) step(par[a]);
    while (i--) step(I[i]);
    if (end) res[qi] = calc();
  }
  return res;
}
```

Numerical (4)

4.1 Polynomials and recurrences

PolyBase.h

Description: A FFT based Polynomial class.

"../number-theory/ModularArithmetic.h", "FastFourierTransform.h", "FastFourierTransformMod.h", "NumberTheoreticTransform.h"

499a15, 35 lines

```
typedef Mod num;
typedef vector<num> poly;
poly &operator+=(poly &a, const poly &b) {
  a.resize(max(sz(a), sz(b)));
  rep(i, 0, sz(b)) a[i] = a[i] + b[i];
  return a;
}
poly &operator-=(poly &a, const poly &b) {
  a.resize(max(sz(a), sz(b)));
  rep(i, 0, sz(b)) a[i] = a[i] - b[i];
  return a;
}

poly &operator*=(poly &a, const poly &b) {
  if (sz(a) + sz(b) < 100){
    poly res(sz(a) + sz(b) - 1);
    rep(i,0,sz(a)) rep(j,0,sz(b))
      res[i + j] = (res[i + j] + a[i] * b[j]);
    return (a = res);
  }
  // auto res = convMod<mod>(vl(all(a)), vl(all(b)));
  auto res = conv(vl(all(a)), vl(all(b)));
  return (a = poly(all(res)));
}
poly operator*(poly a, const num b) {
  poly c = a;
  trav(i, c) i = i * b;
  return c;
}
#define OP(o, oe) \
  poly operator o(poly a, poly b) { \
    poly c = a; \
    return c o##= b; \
  }
OP(*, *=) OP(+, +=) OP(-, -=);
```

PolyInverse.h

Description: A FFT based Polynomial class.

"PolyBase.h"

703c16, 7 lines

```
poly modK(poly a, int k) { return {a.begin(), a.begin() + min(k
    , sz(a))}; }
poly inverse(poly A) {
  poly B = poly({num(1) / A[0]});
  while (sz(B) < sz(A))
    B = modK(B * (poly({num(2)}) - modK(A, 2*sz(B)) * B), 2 *
        sz(B));
  return modK(B, sz(A));
}
```

PolyMod.h

Description: A FFT based Polynomial class.

"PolyBase.h", "PolyInverse.h"

264551, 20 lines

```
poly &operator/=(poly &a, poly b) {
  if (sz(a) < sz(b))
    return a = {};
  int s = sz(a) - sz(b) + 1;
  reverse(all(a)), reverse(all(b));
  a.resize(s), b.resize(s);
  a = a * inverse(b);
  a.resize(s), reverse(all(a));
  return a;
}
OP(/, /=)
poly &operator%=(poly &a, poly &b) {
  if (sz(a) < sz(b))
    return a;
  poly c = (a / b) * b;
  a.resize(sz(b) - 1);
  rep(i, 0, sz(a)) a[i] = a[i] - c[i];
  return a;
}
OP(%, %=)
```

PolyIntegDeriv.h

Description: A FFT based Polynomial class.

"PolyBase.h"

803fd5, 14 lines

```
poly deriv(poly a) {
  if (a.empty()) return {};
  poly b(sz(a) - 1);
  rep(i, 1, sz(a)) b[i - 1] = a[i] * num(i);
  return b;
}
poly integr(poly a) {
  if (a.empty()) return {0};
  poly b(sz(a) + 1);
  b[1] = num(1);
  rep(i, 2, sz(b)) b[i] = b[mod%i]*Mod(-mod/i+mod);
  rep(i, 1 ,sz(b)) b[i] = a[i-1] * b[i];
  return b;
}
```

PolyLogExp.h

Description: A FFT based Polynomial class.

"PolyBase.h", "PolyInverse.h", "PolyIntegDeriv.h"

83ea75, 14 lines

```
poly log(poly a) {
  return modK(integr(deriv(a) * inverse(a)), sz(a));
}
poly exp(poly a) {
  poly b(1, num(1));
  if (a.empty())
    return b;
  while (sz(b) < sz(a)) {
    b.resize(sz(b) * 2);
    b *= (poly({num(1)}) + modK(a, sz(b)) - log(b));
    b.resize(sz(b) / 2 + 1);
  }
  return modK(b, sz(a));
}
```

PolyPow.h

Description: A FFT based Polynomial class.

"PolyBase.h", "PolyLogExp.h"

f0005c, 13 lines

```
poly pow(poly a, ll m) {
  int p = 0, n = sz(a);
  while (p < sz(a) && a[p].v == 0)
    ++p;
  if (ll(m)*p >= sz(a)) return poly(sz(a));
  num j = a[p];
  a = {a.begin() + p, a.end()};
  a = a * (num(1) / j);
  a.resize(n);
  auto res =  exp(log(a) * num(m)) * (j ^ m);
  res.insert(res.begin(), p*m, 0);
  return {res.begin(), res.begin()+n};
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$.
Time: $\mathcal{O}\left(n \log^2 n\right)$

"PolyBase.h", "PolyIntegDeriv.h", "PolyEvaluate.h" b911f5, 11 lines

```
poly interp(vector<num> x, vector<num> y) {
  int n=sz(x);
  vector<poly> up(n*2);
  rep(i,0,n) up[i+n] = poly({num(0)-x[i], num(1)});
  for(int i=n-1; i>0;i--) up[i] = up[2*i]*up[2*i+1];
  vector<num> a = eval(deriv(up[1]), x);
  vector<poly> down(2*n);
  rep(i,0,n) down[i+n] = poly({y[i]*(num(1)/a[i])});
  for(int i=n-1;i>0;i--) down[i] = down[i*2] * up[i*2+1] + down
       [i*2+1] * up[i*2];
  return down[1];
}
```

PolyEvaluate.h

Description: Multi-point evaluation. Evaluates a given polynomial A at $A(x_0), ...A(x_n)$.
Time: $\mathcal{O}\left(n \log^2 n\right)$

"PolyBase.h", "PolyMod.h" dc2cdf, 14 lines

```
vector<num> eval(const poly &a, const vector<num> &x) {
  int n = sz(x);
  if (!n) return {};
  vector<poly> up(2 * n);
  rep(i, 0, n) up[i + n] = poly({num(0) - x[i], 1});
  for (int i = n - 1; i > 0; i--)
    up[i] = up[2 * i] * up[2 * i + 1];
  vector<poly> down(2 * n);
  down[1] = a % up[1];
  rep(i, 2, 2 * n) down[i] = down[i / 2] % up[i];
  vector<num> y(n);
  rep(i, 0, n) y[i] = down[i + n][0];
  return y;
}
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" b00bfe, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}\left(N^2\right)$

"../number-theory/ModPow.h" 96548b, 20 lines

```
vector<ll> berlekampMassey(vector<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;

  ll b = 1;
  rep(i,0,n) { ++m;
    ll d = s[i] % mod;
    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }

  C.resize(L + 1); C.erase(C.begin());
  for (ll& x : C) x = (mod - x) % mod;
  return C;
}
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \dots \geq n-1]$ and $tr[0 \dots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}\left(n^2 \log k\right)$

f4e444, 26 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
  int n = sz(tr);

  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    rep(i,0,n+1) rep(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) rep(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
    res.resize(n + 1);
    return res;
  };

  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;

  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }

  ll res = 0;
  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
}
```

InterpolateFast.h

Description: Find the value of $\sum_{i=1}^{n} i^k$ modulo $10^9 + 7$.
Time: $\mathcal{O}\left(n\right)$

d31fca, 33 lines

```
int interpolate(int x, int k, bool bf = false) {
  if (k == 0) return x;
  // find 1^k + 2^k + ... + x^k
  // (k+1) degree polynomial -> (k+2) points
  if (x <= k + 1 || bf) {
    int s = 0;
    for (int i = 1; i <= x; i ++) {
      s = (s + qpow(i, k)) % mod;
    }
    return s;
  }
  vector<int> pre(k + 2), suf(k + 2), inv(k + 2);
  inv[0] = 1;
  pre[0] = x;
  suf[k + 1] = x - (k + 1);
  for (int i = 1; i <= k; i ++) pre[i] = pre[i - 1] * (x - i) %
       mod; //numerator prefix product
  for (int i = k; i >= 1; i --) suf[i] = suf[i + 1] * (x - i) %
       mod; //numerator suffix product
  for (int i = 1; i <= k + 1; i ++) inv[i] = inv[i - 1] * rv(i)
       % mod; // denominator factorial
  int ans = 0;
  int yi = 0; // 0^k +~ i^k
  int num, denom;
  for (int i = 0; i <= k + 1; i ++) {
    yi = (yi + qpow(i, k)) % mod; // interpolate point: (i, yi)
    if (i == 0) num = suf[1];
    else if (i == k + 1) num = pre[k];
    else num = pre[i - 1] * suf[i + 1] % mod; // numerator
    denom = inv[i] * inv[k + 1 - i] % mod; // denominator
    if ((i + k) & 1) ans += (yi * num % mod) * denom % mod; //
         (-1)^(i-deg) however deg is k+1 here so :)
    else ans -= (yi * num % mod) * denom % mod;
    ans = (ans % mod + mod) % mod;
  }
  return ans;
}
```

4.2 Optimization

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
Usage:
```
vvd A = {{1,-1}, {-1,1}, {-1,-2}};
vd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);
```
Time: $\mathcal{O}\left(NM * \#pivots\right)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}\left(2^n\right)$ in the general case.

aa8530, 68 lines

```
typedef double T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
      rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];}
      rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
```

```
  }

  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      rep(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    }
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      rep(i,0,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                     < MP(D[r][n+1] / D[r][s], B[r])) r = i;
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  }

  T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      rep(i,0,m) if (B[i] == -1) {
        int s = 0;
        rep(j,1,n+1) ltj(D[i]);
        pivot(i, s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

4.3 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}\left(N^3\right)$

bd5cec, 15 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
}
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}\left(N^3\right)$

3313dc, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) rep(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
}
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}\left(n^2m\right)$

44c9ab, 38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

"SolveLinear.h" 08e495, 7 lines

```
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
  rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}\left(n^2m\right)$

fa2d7a, 34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank $<$ n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1})$ (mod p^k) where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}\left(n^3\right)$

ebfff6, 35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
```

```
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

MatrixInverse-mod.h

Description: Invert matrix A modulo a prime. Returns rank; result is stored in A unless singular (rank < n). For prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1})$ (mod p^k) where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

"../number-theory/ModPow.h" a6f68f, 36 lines

```
int matInv(vector<vector<ll>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<ll>> tmp(n, vector<ll>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n) if (A[j][k]) {
      r = j; c = k; goto found;
    }
    return i;
found:
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n) swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c
        ]);
    swap(col[i], col[c]);
    ll v = modpow(A[i][i], mod - 2);
    rep(j,i+1,n) {
      ll f = A[j][i] * v % mod;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] = (A[j][k] - f*A[i][k]) % mod;
      rep(k,0,n) tmp[j][k] = (tmp[j][k] - f*tmp[i][k]) % mod;
    }
    rep(j,i+1,n) A[i][j] = A[i][j] * v % mod;
    rep(j,0,n) tmp[i][j] = tmp[i][j] * v % mod;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    ll v = A[j][i];
    rep(k,0,n) tmp[j][k] = (tmp[j][k] - v*tmp[i][k]) % mod;
  }

  rep(i,0,n) rep(j,0,n)
    A[col[i]][col[j]] = tmp[i][j] % mod + (tmp[i][j] < 0 ? mod
        : 0);
  return n;
}
```

4.4 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: conv(a, b) = c, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$)

97da51, 36 lines

```
typedef complex<double> C;
typedef complex<long double> Cd;
typedef vector<double> vd;
void fft(vector<C>& a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vector<complex<long double>> R(2, 1);
  static vector<C> rt(2, 1);  // (^ 10% faster if double)
  for (static int k = 2; k < n; k *= 2) {
    R.resize(n); rt.resize(n);
    auto x = polar(1.0L, acos(-1.0L) / k);
    rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
  }
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
      a[i + j + k] = a[i + j] - z;
      a[i + j] += z;
    }
}
vd conv(const vd& a, const vd& b) {
  if (a.empty() || b.empty()) return {};
  vd res(sz(a) + sz(b) - 1);
  int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
  vector<C> in(n), out(n);
  copy(all(a), begin(in));
  rep(i,0,sz(b)) in[i].imag(b[i]);
  fft(in);
  for (C& x : in) x *= x;
  rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
  fft(out);
  rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n);
  return res;
}
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" b82773, 22 lines

```
typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  vl res(sz(a) + sz(b) - 1);
  int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M));
  vector<C> L(n), R(n), outs(n), outl(n);
  rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
  rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
  fft(L), fft(R);
  rep(i,0,n) {
    int j = -i & (n - 1);
    outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
    outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
  }
  fft(outl), fft(outs);
  rep(i,0,sz(res)) {
    ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5);
    ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
    res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
  }
  return res;
}
```

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x] g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. conv(a, b) = c, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).
Time: $\mathcal{O}(N \log N)$

"../number-theory/ModPow.h" ced03d, 33 lines

```
const ll mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void ntt(vl &a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vl rt(2, 1);
  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
    rt.resize(n);
    ll z[] = {1, modpow(root, mod >> s)};
    rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
  }
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
      a[i + j + k] = ai - z + (z > ai ? mod : 0);
      ai += (ai + z >= mod ? z - mod : z);
    }
}
vl conv(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s), n = 1
      << B;
  int inv = modpow(n, mod - 2);
  vl L(a), R(b), out(n);
  L.resize(n), R.resize(n);
  ntt(L), ntt(R);
  rep(i,0,n) out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv %
      mod;
  ntt(out);
  return {out.begin(), out.begin() + s};
}
```

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

464cf3, 16 lines

```
void FST(vi& a, bool inv) {
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
```

```
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    }
  }
  if (inv) for (int& x : a) x /= sz(a); // XOR only
}
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  rep(i,0,sz(a)) a[i] *= b[i];
  FST(a, 1); return a;
}
```

Number theory (5)

5.1 Modular arithmetic

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set mod to some number first and then you can use the structure.

3318e2, 18 lines

```
const ll mod = 17; // change to something else
struct Mod {
  ll v;
  Mod() : v(0) {}
  Mod(ll vv) : v(vv % mod) {}
  Mod operator+(Mod b) { return Mod((v + b.v) % mod); }
  Mod operator-(Mod b) { return Mod(v - b.v + mod); }
  Mod operator*(Mod b) { return Mod(v * b.v); }
  Mod operator/(Mod b) { return *this * invert(b); }
  Mod invert(Mod a) { return a^(mod-2); }
  Mod operator^(ll e) {
    ll ans = 1, b = (*this).v;
    for (; e; b = b * b % mod, e /= 2)
      if (e & 1) ans = ans * b % mod;
    return ans;
  }
  explicit operator ll() const { return v; }
};
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

6f684f, 3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h

b83e45, 8 lines

```
const ll mod = 1000000007; // faster if const

ll modpow(ll b, ll e) {
  ll ans = 1;
  for (; e; b = b * b % mod, e /= 2)
    if (e & 1) ans = ans * b % mod;
  return ans;
}
```

ModLog.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod m$, or -1 if no such x exists. modLog(a,1,m) can be used to calculate the order of a.

Time: $\mathcal{O}\left(\sqrt{m}\right)$

c040b8, 11 lines

```
ll modLog(ll a, ll b, ll m) {
  ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<ll, ll> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j;
  if (__gcd(m, e) == __gcd(m, b))
    rep(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.
modsum(to, c, k, m) $= \sum_{i=0}^{\text{to}-1} (ki + c) \% m$. divsum is similar but for floored division.

Time: $\log(m)$, with a large constant.

5c5bc5, 16 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.

Time: $\mathcal{O}(1)$ for modmul, $\mathcal{O}(\log b)$ for modpow

bbbd8f, 11 lines

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans;
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).

Time: $\mathcal{O}\left(\log^2 p\right)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModPow.h" 19a793, 24 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

ShortLucas.h

Description: Lucas' thm: Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod p$. fact and invfact must hold pre-computed factorials / inverse factorials, e.g. from ModInverse.h.

Time: $\mathcal{O}\left(\log_p n\right)$

81845f, 10 lines

```
ll chooseModP(ll n, ll m, int p, vi& fact, vi& invfact) {
  ll c = 1;
  while (n || m) {
    ll a = n % p, b = m % p;
    if (a < b) return 0;
    c = c * fact[a] % p * invfact[b] % p * invfact[a - b] % p;
    n /= p; m /= p;
  }
  return c;
}
```

5.2 Primality

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.

Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" 60dcd1, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" a33cf6, 18 lines

```
ull pollard(ull n) {
  auto f = [n](ull x) { return modmul(x, x, n) + 1; };
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
```

```
  l.insert(l.end(), all(r));
  return l;
}
```

5.3 Divisibility

euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

33ba8f, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
}
```

CRT.h

Description: Chinese Remainder Theorem.

crt(a, m, b, n) computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.

Time: $\log(n)$

"euclid.h" de6b24, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m/g*n : x;
}
```

5.3.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

Description: *Euler's* ϕ function is defined as $\phi(n) := \#$ of positive integers $\le n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} ... (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \le k \le n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$

Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.

Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod p$ $\forall a$.

cf7d6d, 8 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
    for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}
```

5.4 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \ b = k \cdot (2mn), \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

5.5 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.6 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

5.7 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \le m \le n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \le m \le n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.

Time: $\mathcal{O}(n)$

044568, 6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
    use |= 1 << x;                    // (note: minus, not ~!)
  return r;
}
```

6.1.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

6.1.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

6.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod p$.

6.2.3 Binomials

multinomial.h

Description: Computes $\binom{k_1 + \cdots + k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! ... k_n!}$.

a0a312, 6 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
}
```

6.3 General purpose numbers

6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able).
$B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

6.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

6.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

6.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

6.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

6.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max|w_i| < \sim 2^{63}$.
Time: $\mathcal{O}(VE)$ 830a8f, 23 lines

```
const ll inf = LLONG_MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
struct Node { ll dist = inf; int prev = -1; };

void bellmanFord(vector<Node>& nodes, vector<Ed>& eds, int s) {
  nodes[s].dist = 0;
  sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });

  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled vertices
  rep(i,0,lim) for (Ed ed : eds) {
    Node cur = nodes[ed.a], &dest = nodes[ed.b];
    if (abs(cur.dist) == inf) continue;
    ll d = cur.dist + ed.w;
    if (d < dest.dist) {
      dest.prev = ed.a;
      dest.dist = (i < lim-1 ? d : -inf);
    }
  }
  rep(i,0,lim) for (Ed e : eds) {
    if (nodes[e.a].dist == -inf)
      nodes[e.b].dist = -inf;
  }
}
```

7.2 Network flow

MinCostMaxFlow.h

Description: Min-cost max-flow. cap[i][j] != cap[j][i] is allowed; double edges are not. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.
Time: Approximately $\mathcal{O}(E^2)$ fe85cc, 81 lines

```
#include <bits/extc++.h>

const ll INF = numeric_limits<ll>::max() / 4;
typedef vector<ll> VL;

struct MCMF {
  int N;
  vector<vi> ed, red;
  vector<VL> cap, flow, cost;
  vi seen;
  VL dist, pi;
  vector<pii> par;

  MCMF(int N) :
    N(N), ed(N), red(N), cap(N, VL(N)), flow(cap), cost(cap),
    seen(N), dist(N), pi(N), par(N) {}

  void addEdge(int from, int to, ll cap, ll cost) {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
    ed[from].push_back(to);
    red[to].push_back(from);
  }

  void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;

    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({0, s});

    auto relax = [&](int i, ll cap, ll cost, int dir) {
      ll val = di - pi[i] + cost;
      if (cap && val < dist[i]) {
        dist[i] = val;
        par[i] = {s, dir};
        if (its[i] == q.end()) its[i] = q.push({-dist[i], i});
        else q.modify(its[i], {-dist[i], i});
      }
    };

    while (!q.empty()) {
      s = q.top().second; q.pop();
      seen[s] = 1; di = dist[s] + pi[s];
      for (int i : ed[s]) if (!seen[i])
        relax(i, cap[s][i] - flow[s][i], cost[s][i], 1);
      for (int i : red[s]) if (!seen[i])
        relax(i, flow[i][s], -cost[i][s], 0);
    }
    rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
  }

  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
      ll fl = INF;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
        fl = min(fl, r ? cap[p][x] - flow[p][x] : flow[x][p]);
      totflow += fl;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
        if (r) flow[p][x] += fl;
        else flow[x][p] -= fl;
    }
    rep(i,0,N) rep(j,0,N) totcost += cost[i][j] * flow[i][j];
    return {totflow, totcost};
```

```
  }

  // If some costs can be negative, call this before maxflow:
  void setpi(int s) { // (otherwise, leave this out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
      rep(i,0,N) if (pi[i] != INF)
        for (int to : ed[i]) if (cap[i][to])
          if ((v = pi[i] + cost[i][to]) < pi[to])
            pi[to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  }
};
```

EdmondsKarp.h

Description: Flow algorithm with guaranteed complexity $O(VE^2)$. To get edge flow values, compare capacities before and after, and take the positive values only.

482fe0, 35 lines

```
template<class T> T edmondsKarp(vector<unordered_map<int, T>>&
    graph, int source, int sink) {
  assert(source != sink);
  T flow = 0;
  vi par(sz(graph)), q = par;

  for (;;) {
    fill(all(par), -1);
    par[source] = 0;
    int ptr = 1;
    q[0] = source;

    rep(i,0,ptr) {
      int x = q[i];
      for (auto e : graph[x]) {
        if (par[e.first] == -1 && e.second > 0) {
          par[e.first] = x;
          q[ptr++] = e.first;
          if (e.first == sink) goto out;
        }
      }
    }
    return flow;
out:
    T inc = numeric_limits<T>::max();
    for (int y = sink; y != source; y = par[y])
      inc = min(inc, graph[par[y]][y]);

    flow += inc;
    for (int y = sink; y != source; y = par[y]) {
      int p = par[y];
      if ((graph[p][y] -= inc) <= 0) graph[p].erase(y);
      graph[y][p] += inc;
    }
  }
}
```

Dinic.h

Description: Flow algorithm with complexity $O(VE\log U)$ where $U = \max|\text{cap}|$. $O(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $O(\sqrt{V}E)$ for bipartite matching.

d7f0f1, 42 lines

```
struct Dinic {
  struct Edge {
    int to, rev;
    ll c, oc;
    ll flow() { return max(oc - c, 0LL); } // if you need flows
  };
  vi lvl, ptr, q;
  vector<vector<Edge>> adj;
  Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
  void addEdge(int a, int b, ll c, ll rcap = 0) {
    adj[a].push_back({b, sz(adj[b]), c, c});
    adj[b].push_back({a, sz(adj[a]) - 1, rcap, rcap});
  }
  ll dfs(int v, int t, ll f) {
    if (v == t || !f) return f;
    for (int& i = ptr[v]; i < sz(adj[v]); i++) {
      Edge& e = adj[v][i];
      if (lvl[e.to] == lvl[v] + 1)
        if (ll p = dfs(e.to, t, min(f, e.c))) {
          e.c -= p, adj[e.to][e.rev].c += p;
          return p;
        }
    }
    return 0;
  }
  ll calc(int s, int t) {
    ll flow = 0; q[0] = s;
    rep(L,0,31) do { // 'int L=30' maybe faster for random data
      lvl = ptr = vi(sz(q));
      int qi = 0, qe = lvl[s] = 1;
      while (qi < qe && !lvl[t]) {
        int v = q[qi++];
        for (Edge e : adj[v])
          if (!lvl[e.to] && e.c >> (30 - L))
            q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
      }
      while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
    } while (lvl[t]);
    return flow;
  }
  bool leftOfMinCut(int a) { return lvl[a] != 0; }
};
```

MinCut.h

Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s, only traversing edges with positive residual capacity.

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}\left(V^3\right)$

8b0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
  pair<int, vi> best = {INT_MAX, {}};
  int n = sz(mat);
  vector<vi> co(n);
  rep(i,0,n) co[i] = {i};
  rep(ph,1,n) {
    vi w = mat[0];
    size_t s = 0, t = 0;
    rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio. queue
      w[t] = INT_MIN;
      s = t, t = max_element(all(w)) - w.begin();
      rep(i,0,n) w[i] += mat[t][i];
    }
    best = min(best, {w[t] - mat[t][t], co[t]});
    co[s].insert(co[s].end(), all(co[t]));
    rep(i,0,n) mat[s][i] += mat[t][i];
    rep(i,0,n) mat[i][s] = mat[s][i];
    mat[0][t] = INT_MIN;
  }
  return best;
}
```

GomoryHu.h

Description: Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.
Time: $\mathcal{O}\left(V\right)$ Flow Computations

"PushRelabel.h" 0418b3, 13 lines

```
typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
  vector<Edge> tree;
  vi par(N);
  rep(i,1,N) {
    PushRelabel D(N); // Dinic also works
    for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
    tree.push_back({i, par[i], D.calc(i, par[i])});
    rep(j,i+1,N)
      if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
  }
  return tree;
}
```

7.3 Matching

DFSMatching.h

Description: Simple bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: `vi btoa(m, -1); dfsMatching(g, btoa);`
Time: $\mathcal{O}\left(VE\right)$

522b98, 22 lines

```
bool find(int j, vector<vi>& g, vi& btoa, vi& vis) {
  if (btoa[j] == -1) return 1;
  vis[j] = 1; int di = btoa[j];
  for (int e : g[di])
    if (!vis[e] && find(e, g, btoa, vis)) {
      btoa[e] = di;
      return 1;
    }
  return 0;
}
int dfsMatching(vector<vi>& g, vi& btoa) {
  vi vis;
  rep(i,0,sz(g)) {
    vis.assign(sz(btoa), 0);
    for (int j : g[i])
      if (find(j, g, btoa, vis)) {
        btoa[j] = i;
        break;
      }
  }
  return sz(btoa) - (int)count(all(btoa), -1);
}
```

Blossom.h

Description: Edmond's Blossom general Matching, Best known time is $O(V^2\sqrt{V})$ Use with care
Time: $\mathcal{O}\left(V^3\right)$

2d3db1, 55 lines

```
struct Edmonds {
  int n, T;
  vector<vi> edge;
  vi mate, p, vis, base, toJoin;
  Edmonds(int N) : n(N), T(0), edge(n), mate(n, -1), p(n), vis(
      n), base(n) { }
  void add(int a, int b) { edge[a].pb(b); edge[b].pb(a); }
  int getBase(int i) { return base[i] == i ? i : (base[i] =
      getBase(base[i])); }
  void mark_path(int pos, int nx, int b, vi &path) {
    for (; getBase(pos) != b; pos = p[nx]) {
      p[pos] = nx, nx = mate[pos];
```

```
      toJoin.pb(pos); toJoin.pb(nx);
      if (!vis[nx]) vis[nx] = ++T, path.pb(nx);
    }
  }
  bool go(int pos) {
    for (int nx : edge[pos]) {
      int b, bpos = getBase(pos), bnx = getBase(nx);
      if (bpos == bnx) continue;
      else if (vis[nx]) {
        vi path; toJoin.clear();
        if (vis[bnx] < vis[bpos]) mark_path(pos, nx, b = bnx,
            path);
        else mark_path(nx, pos, b = bpos, path);
        for (int z : toJoin) base[getBase(z)] = b;
        for (int z : path) if (go(z)) return 1;
      } else if (p[nx] == -1) {
        p[nx] = pos;
        if (mate[nx] == -1) {
          for (int y; nx != -1; nx = pos)
            y = p[nx], pos = mate[y], mate[nx] = y, mate[y] =
                nx;
          return 1;
        }
        if (!vis[mate[nx]]) { vis[mate[nx]] = ++T; if (go(mate[
            nx])) return 1; }
      }
    }
    return 0;
  }
  void init_dfs() { rep(i, 0, n) vis[i] = 0, p[i] = -1, base[i]
      = i; }
  bool dfs(int root) {vis[root] = ++T; return go(root); }
  void match() {
    int ans = 0;
    for (int pos = 0; pos < n; pos++) {
      if (mate[pos] != -1) continue;
      for (int nx : edge[pos]) {
        if (mate[nx] == -1) {
          mate[pos] = nx; mate[nx] = pos; ans++; break;
        }
      }
    }
    init_dfs();
    rep(i, 0, n) if (mate[i] == -1 && dfs(i)) ans++, init_dfs()
        ;
    cout << ans * 2 << endl;
    rep(i, 0, n) if (i < mate[i])
      cout << i + 1 << " " << mate[i] + 1 << " \n"[i == n - 1];
  }
};
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.

Time: $\mathcal{O}\left(N^2M\right)$ 1e0fe9, 31 lines

```
pair<int, vi> hungarian(const vector<vi> &a) {
  if (a.empty()) return {0, {}};
  int n = sz(a) + 1, m = sz(a[0]) + 1;
  vi u(n), v(m), p(m), ans(n - 1);
  rep(i,1,n) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vi dist(m, INT_MAX), pre(m, -1);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1, delta = INT_MAX;
      rep(j,1,m) if (!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      }
      rep(j,0,m) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    }
  }
  rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
}
```

7.4 DFS algorithms

SCCTarjan.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.

Usage: scc(graph, [&](vi& v) { ... }) visits all components in reverse topological order. comp[i] holds the component index of a node (a component only has edges to components with lower index). ncomps will contain the number of components.

Time: $\mathcal{O}(E + V)$ 17ec58, 15 lines

```
void tarjan(LL pos) {
  path.pb(pos); vis[pos] = 1; low[pos] = num[pos] = ++curtime;
  for (int i = 0; i < edge[pos].size(); i++) {
    LL nx = edge[pos][i];
    if (!num[nx]) tarjan(nx);
    if (vis[nx]) low[pos] = min(low[pos], low[nx]);
  }
  if (low[pos] == num[pos]) { // Push path.back() until equal
      pos }
}

int main() {
  for (int i = 1; i <= n; i++) {
    if (num[i] == 0) tarjan(i);
  }
}
```

Tarjan.h

Description: Finds AP and Bridge.

Time: $\mathcal{O}(E + V)$ 50f160, 23 lines

```
void tarjan(LL pos, LL par) {
  low[pos] = num[pos] = ++dfscnt;
  trav(nx, edge[pos]){
    if (nx == par) continue;
    if (!num[nx]) {
      if (pos == currroot) curchild++;
      tarjan(nx, pos);
      if (low[nx] >= num[pos]) isArticulationPoint[pos] = 1;
      // if(low[nx] > num[pos]) isBridge[pos][nx] = 1;
      low[pos] = min(low[pos], low[nx]);
    } else low[pos] = min(low[pos], num[nx]);
  }
}

int main() {
  for (int i = 1; i <= n; i++) {
    if (num[i]) continue;
    currroot = i;
    curchild = 0;
    tarjan(i, -1);
    isArticulationPoint[i] = (curchild > 1);
  }
}
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.

Time: $\mathcal{O}(V + E)$ 780b64, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
  int n = sz(gr);
  vi D(n), its(n), eu(nedges), ret, s = {src};
  D[src]++; // to allow Euler paths, not just cycles
  while (!s.empty()) {
    int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
    if (it == end){ ret.push_back(x); s.pop_back(); continue; }
    tie(y, e) = gr[x][it++];
    if (!eu[e]) {
      D[x]--, D[y]++;
      eu[e] = 1; s.push_back(y);
    }}
  for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
  return {ret.rbegin(), ret.rend()};
}
```

7.5 Coloring

CycleOfLengthK.h

Description: Find cycle of length K = 3, 4;

Time: $\mathcal{O}(V \log V)$ in planar graph, $\mathcal{O}\left(E^{3/2}\right)$ in dense graph 9a5a9f, 28 lines

```
pll cycle(const vector<pair<int, int>> &edges) {
  int n = 0, i;
  for (auto [u, v] : edges) n = max({n, u, v});
  ++n;
  vector d(n, 0), id(d), rk(d), cnt(d);
  vector e(n, vector(0, 0)), lk(n, vector(0, 0));
  for (auto [u, v] : edges) ++d[u], ++d[v];
  iota(all(id), 0); sort(all(id), [&](int x, int y) { return d[
      x] < d[y]; });
  for (i = 0; i < n; i++) rk[id[i]] = i;
  for (auto [u, v] : edges) {
    if (rk[u] > rk[v]) swap(u, v);
    e[u].push_back(v);
    lk[u].push_back(v);
    lk[v].push_back(u);
  }
  ll c3 = 0;
  for (i = 0; i < n; i++) {
    for (int u : e[i]) cnt[u] = 1;
    for (int u : e[i]) for (int v : e[u]) c3 += cnt[v];
    for (int u : e[i]) cnt[u] = 0;
  }
  ll c4 = 0;
  for (i = 0; i < n; i++) {
    for (int u : lk[i]) for (int v : e[u]) if (rk[v] > rk[i])
        c4 += cnt[v]++;
    for (int u : lk[i]) for (int v : e[u]) cnt[v] = 0;
  }
  return {c3, c4};
}
```

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)
Time: $\mathcal{O}(NM)$

e210e2, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {
  vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
  for (pii e : eds) ++cc[e.first], ++cc[e.second];
  int u, v, ncols = *max_element(all(cc)) + 1;
  vector<vi> adj(N, vi(ncols, -1));
  for (pii e : eds) {
    tie(u, v) = e;
    fan[0] = v;
    loc.assign(ncols, 0);
    int at = u, end = u, d, c = free[u], ind = 0, i = 0;
    while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
      loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
    cc[loc[d]] = c;
    for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
      swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
    while (adj[fan[i]][d] != -1) {
      int left = fan[i], right = fan[++i], e = cc[i];
      adj[u][e] = left;
      adj[left][e] = u;
      adj[right][e] = -1;
      free[right] = e;
    }
    adj[u][d] = fan[i];
    adj[fan[i]][d] = u;
    for (int y : {fan[0], u, end})
      for (int& z = free[y] = 0; adj[y][z] != -1; z++);
  }
  rep(i,0,sz(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
  return ret;
}
```

VertexColoring.h

Description: Calculates the chromatic number of an undirected graph.
Time: $\mathcal{O}(2.4422^n)$

"MaximalCliques.h" 011734, 17 lines

```
const int MAXN = 15;
int memo[1<<MAXN] = {0};
int aux(vector<B>& eds, B nodes) {
  auto k = nodes.to_ulong();
  if (!nodes.any()) return 0;
  if (memo[k]) return memo[k];
  int r = MAXN;
  cliques(eds, [&](B x) {
    r = min(r, 1 + aux(eds, nodes & ~x));
  }, nodes);
  return memo[k] = r;
}
int chromaticNumber(vector<B>& eds) {
  vector<B> comp;
  rep(i, 0, sz(eds)) comp.push_back((~eds[i]).reset(i));
  return aux(comp, (1 << sz(comp)) - 1);
}
```

7.6 Heuristics

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.
Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

b0d5b1, 12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

MaximumClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.
Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

f7c0bc, 49 lines

```
typedef vector<bitset<200>> vb;
struct Maxclique {
  double limit=0.025, pk=0;
  struct Vertex { int i, d=0; };
  typedef vector<Vertex> vv;
  vb e;
  vv V;
  vector<vi> C;
  vi qmax, q, S, old;
  void init(vv& r) {
    for (auto& v : r) v.d = 0;
    for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
    sort(all(r), [](auto a, auto b) { return a.d > b.d; });
    int mxD = r[0].d;
    rep(i,0,sz(r)) r[i].d = min(i, mxD) + 1;
  }
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (sz(R)) {
      if (sz(q) + R.back().d <= sz(qmax)) return;
      q.push_back(R.back().i);
      vv T;
      for(auto v:R) if (e[R.back().i][v.i]) T.push_back({v.i});
      if (sz(T)) {
        if (S[lev]++ / ++pk < limit) init(T);
        int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
        C[1].clear(), C[2].clear();
        for (auto v : T) {
          int k = 1;
          auto f = [&](int i) { return e[v.i][i]; };
          while (any_of(all(C[k]), f)) k++;
          if (k > mxk) mxk = k, C[mxk + 1].clear();
          if (k < mnk) T[j++].i = v.i;
          C[k].push_back(v.i);
        }
        if (j > 0) T[j - 1].d = 0;
        rep(k,mnk,mxk + 1) for (int i : C[k])
          T[j].i = i, T[j++].d = k;
        expand(T, lev + 1);
      } else if (sz(q) > sz(qmax)) qmax = q;
      q.pop_back(), R.pop_back();
    }
  }
  vi maxClique() { init(V), expand(V); return qmax; }
  Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) {
    rep(i,0,sz(e)) V.push_back({i});
  }
};
```

MaximumIndependentSet.h

Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.

7.7 Trees

LinkCutTree.h

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.
Time: All operations take amortized $\mathcal{O}(\log N)$.

5909e2, 90 lines

```
struct Node { // Splay tree. Root's pp contains tree's parent.
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted)
  }
  void pushFlip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
  }
  int up() { return p ? p->c[1] == this : -1; }
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y : x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      z->c[h ^ 1] = b ? x : this;
    }
    y->c[i ^ 1] = b ? this : x;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
  }
  void splay() {
    for (pushFlip(); p; ) {
      if (p->p) p->p->pushFlip();
      p->pushFlip(); pushFlip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);
    }
  }
  Node* first() {
    pushFlip();
    return c[0] ? c[0]->first() : (splay(), this);
  }
};

struct LinkCut {
  vector<Node> node;
  LinkCut(int N) : node(N) {}

  void link(int u, int v) { // add an edge (u, v)
    assert(!connected(u, v));
    makeRoot(&node[u]);
    node[u].pp = &node[v];
  }
  void cut(int u, int v) { // remove an edge (u, v)
    Node *x = &node[u], *top = &node[v];
    makeRoot(top); x->splay();
```

```
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;
      x->fix();
    }
  }
  bool connected(int u, int v) { // are u, v in the same tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();
  }
  void makeRoot(Node* u) {
    access(u);
    u->splay();
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;
      u->fix();
    }
  }
  Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; }
      pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
  }
};
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.

Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 39e620, 60 lines

```
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
  RollbackUF uf(n);
  vector<Node*> heap(n);
  for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1,-1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  rep(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) {
        Node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      }
    }
    rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
  }

  for (auto& [u,t,comp] : cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  }
  rep(i,0,n) par[i] = in[i].a;
  return {res, par};
}
```

7.8 Math

7.8.1 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--, mat[b][b]++` (and `mat[b][a]--, mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

7.8.2 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

Geometry (8)

8.1 Geometric primitives

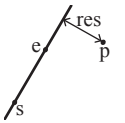
Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.) 47ec0a, 28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
};
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

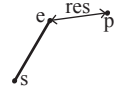


"Point.h" f6bf6b, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.



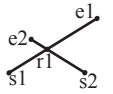
Usage:
```
Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;
```

"Point.h" 5c88f4, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



Usage:
```
vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;
```

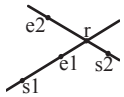
"Point.h", "OnSegment.h" 9d57f2, 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
```

```
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
}
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage: `auto res = lineInter(s1,e1,s2,e2);`

```
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
```

"Point.h" a01f81, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 3af81c, 9 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

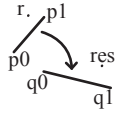
"Point.h" c597e8, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h

Description:

Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



"Point.h" 03a306, 6 lines

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab insted. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h" b5562d, 5 lines

```
template<class P>
P lineProj(P a, P b, P p, bool refl=false) {
  P v = b - a;
  return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
}
```

RotationalSwap.h

Description: Find whether exists triangle whose area is exactly S.

2478a0, 85 lines

```
struct point {
  ll x, y;
  point() {}
  point(ll _x, ll _y) {
    x = _x, y = _y;
  }
};
ll cross(point a, point b) {
  return a.x * b.y - a.y * b.x;
}
ll area(point p, point q, point r) {
  return abs(cross(p, q) + cross(q, r) + cross(r, p));
}
struct seg {
  int i, j;
  point a, b, vec;
  seg() {}
  seg(int _i, int _j, point _a, point _b, point _vec) {
    i = _i, j = _j, a = _a, b = _b, vec = _vec;
  };
};
const int N = 2005;
ll n, s;
point arr[N];
bool cmp(point a, point b) {
  if (a.y == b.y) return a.x < b.x;
  return a.y < b.y;
}
bool cmp2(seg a, seg b) {
  return cross(a.vec, b.vec) > 0;
}
vector<seg> segment;
int ranked[N], id[N];
int main() {
  scanf("%lld %lld", &n, &s); s *= 2;
  for (int i = 1; i <= n; i++) {
    scanf("%lld %lld", &arr[i].x, &arr[i].y);
  }
  sort(arr + 1, arr + n + 1, cmp);
  for (int i = 1; i <= n; i++) for (int j = 1; j < i; j++) {
      segment.push_back(seg(i, j, arr[i], arr[j], point(arr[i].
          x - arr[j].x, arr[i].y - arr[j].y)));
    }
  sort(segment.begin(), segment.end(), cmp2);
  for (int i = 1; i <= n; i++) ranked[i] = id[i] = i;
  for (auto cur : segment) {
    int a = id[cur.j], b = id[cur.i];
    int l = 1, r = a - 1;
    while (l <= r) {
      int mid = (l + r) >> 1;
      ll hasil = area(arr[ranked[mid]], cur.a, cur.b);
      if (hasil == s) {
        printf("Yes\n");
        printf("%lld %lld\n", arr[ranked[mid]].x, arr[ranked[
            mid]].y);
        printf("%lld %lld\n", cur.a.x, cur.a.y);
        printf("%lld %lld\n", cur.b.x, cur.b.y);
        return 0;
      } else if (hasil < s) {
        r = mid - 1;
      } else {
        l = mid + 1;
      }
    }
    l = b + 1, r = n;
    while (l <= r) {
      int mid = (l + r) >> 1;
      ll hasil = area(arr[ranked[mid]], cur.a, cur.b);
      if (hasil == s) {
        printf("Yes\n");
        printf("%lld %lld\n", arr[ranked[mid]].x, arr[ranked[
            mid]].y);
        printf("%lld %lld\n", cur.a.x, cur.a.y);
        printf("%lld %lld\n", cur.b.x, cur.b.y);
        return 0;
      } else if (hasil > s) {
        r = mid - 1;
      } else {
        l = mid + 1;
      }
    }
    assert(a + 1 == b);
    swap(ranked[a], ranked[b]);
    swap(id[cur.i], id[cur.j]);
  }
  printf("No\n");
  return 0;
}
```

RotationalSweep.h

Description: Codechef blue red

Time: $\mathcal{O}\left(N^2 \log N\right)$

"Point.h" 67e3bc, 40 lines

```
template<class P>
P RotationalSweep(vector<P> &all) {
  for (int i = 0; i < n; i++) {
    vector <PP> kiri, kanan;
    for (int j = 0; j < n; j++) {
      PP curtmp = all[j];
      curtmp.center = curtmp.center - all[i].center;
      if (i == j) continue;
      if (curtmp.center.y >= 0) kiri.pb(curtmp);
      else kanan.pb(curtmp);
    }
    sort(kiri.begin(), kiri.end());
    sort(kanan.begin(), kanan.end());
    kiri.insert(kiri.end(), kanan.begin(), kanan.end());
    LL cnt[2] = {0, 0};
    LL N = kiri.size();
    LL hi = 0;
    cnt[kiri[0].color]++;
    for (int j = 0; j < N; j++) {
      do {
        LL nx = (hi + 1) % N;
        if (pivot.ccw(kiri[j].center, kiri[nx].center) <= 0)
            break;
```

```
        cnt[kiri[nx].color]++;
        hi = nx;
      } while (1);
      LL curcnt[2] = {initialColor[0] - cnt[0], initialColor[1]
          - cnt[1]};
      curcnt[all[i].color]--;
      cnt[kiri[j].color]--;
      ans = min(ans, cnt[0] + curcnt[1]);
      ans = min(ans, cnt[1] + curcnt[0]);
      cnt[kiri[j].color]++;
      if (hi == j) {
        LL nx = (hi + 1) % N;
        cnt[kiri[nx].color]++;
        hi = nx;
      }
      cnt[kiri[j].color]--;
    }
  }
}
```

8.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 84d6d3, 11 lines

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" b0153d, 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" e0cfba, 9 lines

```
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
  P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
  double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
  if (h2 < 0) return {};
  if (h2 == 0) return {p};
  P h = ab.unit() * sqrt(h2);
  return {p - h, p + h};
}
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.

Time: $\mathcal{O}(n)$

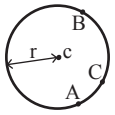
"../../content/geometry/Point.h" a1ee63, 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  };
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
}
```

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 1caa3a, 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.

Time: expected $\mathcal{O}(n)$

"circumcircle.h" 09dd0a, 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
}
```

8.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`

Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 2bf504, 11 lines

```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  }
  return cnt;
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f12300, 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
}
```

PolygonCenter.h

Description: Returns the center of mass for a polygon.

Time: $\mathcal{O}(n)$

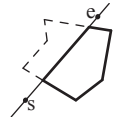
"Point.h" 9706dc, 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

PolygonCut.h

Description:

Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.



Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h", "lineIntersection.h" f2b7d4, 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
```

```
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

PolygonUnion.h

Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)

Time: $\mathcal{O}\left(N^2\right)$, where N is the total number of points

"Point.h", "sideOf.h" 3931c6, 33 lines

```
typedef Point<double> P;
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y; }
double polyUnion(vector<vector<P>>& poly) {
  double ret = 0;
  rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
    P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
    vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
    rep(j,0,sz(poly)) if (i != j) {
      rep(u,0,sz(poly[j])) {
        P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])];
        int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
        if (sc != sd) {
          double sa = C.cross(D, A), sb = C.cross(D, B);
          if (min(sc, sd) < 0)
            segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
        } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))>0){
          segs.emplace_back(rat(C - A, B - A), 1);
          segs.emplace_back(rat(D - A, B - A), -1);
        }
      }
    }
    sort(all(segs));
    for (auto& s : segs) s.first = min(max(s.first, 0.0), 1.0);
    double sum = 0;
    int cnt = segs[0].second;
    rep(j,1,sz(segs)) {
      if (!cnt) sum += segs[j].first - segs[j - 1].first;
      cnt += segs[j].second;
    }
    ret += A.cross(B) * sum;
  }
  return ret / 2;
}
```

ConvexHull.h

Description:
Returns a vector of the points of the convex hull in counterclockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Time: $\mathcal{O}\left(n \log n\right)$

"Point.h" 310954, 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(all(pts)))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

onionDecomposition.h

Description: Online Dynamic Convex Hull

<bits/stdc++.h> 130a28, 190 lines

```
using namespace std;
using ll = int64_t;

// Decremental convex hull in O(n log n)
// From "Applications of a semi-dynamic convex hull algorithm"
//   by J. Hershberger and S. Suri
struct Upper_Hull{
  struct Link{
    Point p;
    Link *prev = nullptr, *next = nullptr;
    int id;
  };
  struct Node{
    Link *chain, *chain_back, *tangent;
  };
  template<typename S, typename T>
  pair<Link*, Link*> find_bridge(Link*l, Link*r, S next, T
      convex){
    while(next(l) || next(r)){
      if(!next(r) || (next(l) && convex(Point{0, 0}, next(l)->p
          - l->p, next(r)->p - r->p))){
        if(convex(l->p, next(l)->p, r->p)) l = next(l);
        else break;
      } else {
        if(!convex(l->p, r->p, next(r)->p)) r = next(r);
        else break;
      }
    }
    return {l, r};
  }
  template<bool rev = false>
  void fix_chain(int u, Link*l, Link*r){
    if(rev){ // l and r to the right of actual bridge
      tie(r, l) = find_bridge(r, l,
        [](Link*x){ return x->prev; },
        [](Point const&a, Point const&b, Point const&c){
          return ccw(a, b, c) >= 0;
        });
    } else { // l and r to the left of actual bridge
      tie(l, r) = find_bridge(l, r,
        [](Link*x){ return x->next; },
        [](Point const&a, Point const&b, Point const&c){
          return ccw(a, b, c) <= 0;
        });
    }
    tree[u].tangent = l;
    tree[u].chain = tree[2*u].chain;
    tree[u].chain_back = tree[2*u+1].chain_back;
    tree[2*u].chain = l->next;
    tree[2*u+1].chain_back = r->prev;
    if(l->next) l->next->prev = nullptr;
    else tree[2*u].chain_back = nullptr;
    if(r->prev) r->prev->next = nullptr;
    else tree[2*u+1].chain = nullptr;
    l->next = r; r->prev = l;
  }
  void build(int u, int a, int b){
    if(b-a == 1){
      tree[u].chain = tree[u].chain_back = &lists[a];
      tree[u].tangent = nullptr;
      return;
    }
    const int m = a + (b-a)/2;
    build(2*u, a, m); build(2*u+1, m, b);
    auto l = tree[2*u].chain, r = tree[2*u+1].chain;
    fix_chain(u, l, r);
  }
  void rob(int u, int v){
    tree[u].chain = tree[v].chain;
    tree[v].chain = nullptr;
    tree[u].chain_back = tree[v].chain_back;
    tree[v].chain_back = nullptr;
  }
  void remove(int u, int a, int b, int const&i){
    if(i < a || i >= b) return;
    // we should never hit a leaf
    assert(b-a > 1);
    const int m = a + (b-a)/2;
    // one child -> that child contains i
    if(!tree[u].tangent){
      int v = i<m ? 2*u : 2*u+1;
      tree[v].chain = tree[u].chain;
      tree[v].chain_back = tree[u].chain_back;
      if(i < m) remove(2*u, a, m, i);
      else remove(2*u+1, m, b, i);
      rob(u, v);
      return;
    }
    // restore hull of children
    auto l = tree[u].tangent, r = l->next;
    l->next = tree[2*u].chain;
    if(tree[2*u].chain) tree[2*u].chain->prev = l;
    else tree[2*u].chain_back = l;
    tree[2*u].chain = tree[u].chain;
    r->prev = tree[2*u+1].chain_back;
    if(tree[2*u+1].chain_back)
      tree[2*u+1].chain_back->next = r;
    else tree[2*u+1].chain = r;
    tree[2*u+1].chain_back = tree[u].chain_back;
    // delete i
    const int v = i<m ? 2*u : 2*u+1;
    // only i
    if(tree[v].chain == tree[v].chain_back && tree[v].chain->id
        == i){
      tree[v].chain = tree[v].chain_back = nullptr;
      rob(u, v^1);
      tree[u].tangent = nullptr;
      return;
    }
    if(i < m){
      if(l->id == i) l = l->next;
      remove(2*u, a, m, i);
      if(!l) l = tree[2*u].chain_back;
      fix_chain<true>(u, l, r);
    } else {
      if(r->id == i) r = r->prev;
      remove(2*u+1, m, b, i);
      if(!r) r = tree[2*u+1].chain;
      fix_chain<false>(u, l, r);
    }
  }
  void remove(int i){
    // i is the only point
    if(tree[1].chain == tree[1].chain_back){
      tree[1].chain = tree[1].chain_back = nullptr;
      return;
    }
    remove(1, 0, n, i);
  }
  Upper_Hull(vector<Point> const&v) : n(v.size()), tree(4*n),
      lists(n){
    assert(is_sorted(v.begin(), v.end()));
    for(int i=0; i<n; ++i){
```

```
      lists[i].p = v[i];
      lists[i].id = i;
    }
    build(1, 0, n);
  }
  vector<int> get_hull(){
    vector<int> ret;
    for(Link* u = tree[1].chain; u; u=u->next)
      ret.push_back(u->id);
    return ret;
  }
  vector<Point> get_hull_points(){
    vector<Point> ret;
    for(Link* u = tree[1].chain; u; u=u->next)
      ret.push_back(u->p);
    return ret;
  }
  int n;
  vector<Node> tree;
  vector<Link> lists;
};

// test code from https://codeforces.com/blog/entry/75929
signed main(){
  int N;
  scanf("%d",&N);
  vector<int> layer(N);
  vector<int> ans(N);
  vector<Point> ps;
  map<Point,int> id;
  for(int i=0;i<N;i++){
    int X,Y;
    scanf("%d %d",&X,&Y);
    ps.push_back({X,Y});
    id[{X,Y}]=i;
  }
  sort(ps.begin(),ps.end());
  Upper_Hull left(ps);
  reverse(ps.begin(),ps.end());
  for(auto& p:ps) p=-p;
  Upper_Hull right(ps);
  for(auto& p:ps) p=-p;
  reverse(ps.begin(),ps.end());
  for(int l=1,cnt=0;cnt<N;l++){
    set<int> hull;
    for(int i:left.get_hull()) hull.insert(i);
    for(int i:right.get_hull()) hull.insert(N-1-i);
    for(int i:hull){
      assert(!layer[i]);
      cnt++;
      layer[i]=l;
      left.remove(i);
      right.remove(N-1-i);
    }
  }
  for(int i=0;i<N;i++) ans[id[ps[i]]]=layer[i];
  for(int i=0;i<N;i++) printf("%d\n",ans[i]);
  return 0;
}
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: $\mathcal{O}(n)$

"Point.h" c571b8, 12 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    }
  return res.second;
}
```

HalfPlane.h

Description: Computes the intersection of a set of half-planes. Input is given as a set of planes, facing left. Output is the convex polygon representing the intersection. The points may have duplicates and be collinear. Will not fail catastrophically if 'eps > sqrt(2)(line intersection error)'. Likely to work for more ranges if 3 half planes are never guaranteed to intersect at the same point.
Time: $\mathcal{O}(n \log n)$

"Point.h", "sideOf.h", "lineIntersection.h" eda44b, 31 lines

```
typedef Point<double> P;
typedef array<P, 2> Line;
#define sp(a) a[0], a[1]
#define ang(a) (a[1] - a[0]).angle()

int angDiff(Line a, Line b) { return sgn(ang(a) - ang(b)); }
bool cmp(Line a, Line b) {
  int s = angDiff(a, b);
  return (s ? s : sideOf(sp(a), b[0])) < 0;
}
vector<P> halfPlaneIntersection(vector<Line> vs) {
  const double EPS = sqrt(2) * 1e-8;
  sort(all(vs), cmp);
  vector<Line> deq(sz(vs) + 5);
  vector<P> ans(sz(vs) + 5);
  deq[0] = vs[0];
  int ah = 0, at = 0, n = sz(vs);
  rep(i,1,n+1) {
    if (i == n) vs.push_back(deq[ah]);
    if (angDiff(vs[i], vs[i - 1]) == 0) continue;
    while (ah<at && sideOf(sp(vs[i]), ans[at-1], EPS) < 0)
      at--;
    while (i!=n && ah<at && sideOf(sp(vs[i]),ans[ah],EPS)<0)
      ah++;
    auto res = lineInter(sp(vs[i]), sp(deq[at]));
    if (res.first != 1) continue;
    ans[at++] = res.second, deq[at] = vs[i];
  }
  if (at - ah <= 2) return {};
  return {ans.begin() + ah, ans.begin() + at};
}
```

HalfplaneSet.h

Description: Data structure that dynamically keeps track of the intersection of halfplanes.

<bits/stdc++.h> ff9da9, 95 lines

```
using namespace std;
using T = int;
using T2 = long long;
using T4 = __int128_t;
const T2 INF = 2e9;
struct Line { T a, b; T2 c; };
bool operator<(Line m, Line n) {
  auto half = [&](Line m) {
    return m.b < 0 || m.b == 0 && m.a < 0; };
  return make_tuple(half(m), (T2)m.b * n.a) <
    make_tuple(half(n), (T2)m.a * n.b);
}
tuple<T4, T4, T2> LineIntersection(Line m, Line n) {
  T2 d = (T2)m.a * n.b - (T2)m.b * n.a; // assert(d);
  T4 x = (T4)m.c * n.b - (T4)m.b * n.c;
  T4 y = (T4)m.a * n.c - (T4)m.c * n.a;
  return {x, y, d};
}
Line LineFromPoints(T x1, T y1, T x2, T y2) {
  T a = y1 - y2, b = x2 - x1;
  T2 c = (T2)a * x1 + (T2)b * y1;
  return {a, b, c};
}
ostream& operator<<(ostream& out, Line l) {
  out << "(" << l.a << " * x + " << l.b << " * y <= " << l.c <<
      ")";
  return out;
}
struct HalfplaneSet : multiset<Line> {
  HalfplaneSet() {
    insert({+1, 0, INF}); insert({0, +1, INF});
    insert({-1, 0, INF}); insert({0, -1, INF});
  };

  auto prv(auto it) { return --(it == begin() ? end() : it); }
  auto nxt(auto it) { return (++it == end() ? begin() : it); }
  bool bad(auto it) {
    auto l = *it, pl = *prv(it), nl = *nxt(it);
    T4 x, y; T2 d; tie(x, y, d) = LineIntersection(pl, nl);
    // auto [x, y, d] = LineIntersection(pl, nl);
    T4 sat = l.a * x + l.b * y - (T4)l.c * d;
    if (d < 0 && sat < 0) {
      clear(); // infeasible
    }
    return d > 0 && sat <= 0 || d == 0 && sat < 0;
  }
  void Cut(Line l) { // add ax + by <= c
    if (empty()) return;
    auto it = insert(l);
    if (bad(it)) { erase(it); return; }
    while (size()) {
      auto nit = nxt(it);
      if (bad(nit)) erase(nit);
      else break;
    }
    while (size()) {
      auto pit = prv(it);
      if (bad(pit)) erase(pit);
      else break;
    }
  }
  double Maximize(T a, T b) { // max ax + by
    if (empty()) return -1/0.;
    auto it = lower_bound({-b, a});
    if (it == end()) it = begin();
    // auto [x, y, d] = LineIntersection(*prv(it), *it);
    // return (1.0 * a * x + 1.0 * b * y) / d;
  }
  double Area() {
    long double total = 0.;
    for (auto it = begin(); it != end(); ++it) {
      T4 x1, y1; T2 d1; tie(x1, y1, d1) = LineIntersection(*prv
          (it), *it);
      T4 x2, y2; T2 d2; tie(x2, y2, d2) = LineIntersection(*it,
          *nxt(it));
    //   auto [x1, y1, d1] = LineIntersection(*prv(it), *it);
    //   auto [x2, y2, d2] = LineIntersection(*it, *nxt(it));
      total += (1.0L * x1/d1 * y2/d2 - 1.0L * x2/d2 * y1/d1);
    }
    return total * 0.5L;
  }
};
int main() {
```

```
  //ifstream cin("camera.in");
  //ofstream cout("camera.out");
  int t; cin >> t;
  while (t--) {
    int n; cin >> n;
    vector<T> x(n), y(n);
    for (int i = 0; i < n; ++i)
      cin >> x[i] >> y[i];
    HalfplaneSet HS;
    for (int j = n - 1, i = 0; i < n; j = i++)
      HS.Cut(LineFromPoints(x[j], y[j], x[i], y[i]));
    cout << fixed << setprecision(6) << HS.Area() / 4e14 << '\n
        ';
  }
  return 0;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 14 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: $\bullet$ $(-1,-1)$ if no collision, $\bullet$ $(i,-1)$ if touching the corner i, $\bullet$ (i,i) if along side $(i,i+1)$, $\bullet$ (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

"Point.h" 7cf45b, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  }
  return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

PolygonStab.h

Description: Stab the polygon isi, with line a, b, find the longest segment with valid stab
Time: $\mathcal{O}\left(N^2 \log N\right)$

"Point.h" 5aa689, 26 lines

```
P isi[1005];
template<class P>
P PolygonStab(P a, P b) {
  vector<intersectPoint> all;
  for (int k = 1; k <= n; k++) {
    auto res = lineInter(a, b, isi[k], isi[k + 1 <= n ? k + 1 :
        1]);
    if (res.fi == 1 && onSegment(isi[k], isi[k + 1 <= n ? k + 1
        : 1], res.se)) {
      all.pb({res.se, {a.ccw(b, isi[k]), a.ccw(b, isi[k + 1 <=
          n ? k + 1 : 1])}});
    }
  }
  sort(all.begin(), all.end());
  int isInside = -1, pre;
  LD curAns = 0; P lst;
  for (int k = 0; k < all.size(); k++) {
    if (isInside >= 0) curAns += (all[k].fi - all[k - 1].fi).
        dist();
    if (all[k].se.fi * all[k].se.se == -1) isInside = -isInside
        ;
    else if (isInside == 0) isInside = (all[k].se.fi + all[k].
        se.se) * pre;
    else {
      pre = (all[k].se.fi + all[k].se.se) * isInside;
      isInside = 0;
    }
    ans = doubleMax(ans, curAns);
    if (isInside == -1) curAns = 0, lst = all[k + 1 <= (int)(
        all.size()) - 1 ? k + 1 : 0].fi;
  }
  return curAns;
}
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" ac41a6, 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
}
```

ManhattanMST.h

Description: Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights w(p, q) = —p.x - q.x— + —p.y - q.y—. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
Time: $\mathcal{O}(N \log N)$

"Point.h" df6f59, 23 lines

```
typedef Point<int> P;
vector<array<int, 3>> manhattanMST(vector<P> ps) {
  vi id(sz(ps));
  iota(all(id), 0);
  vector<array<int, 3>> edges;
  rep(k,0,4) {
    sort(all(id), [&](int i, int j) {
        return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
    map<int, int> sweep;
    for (int i : id) {
      for (auto it = sweep.lower_bound(-ps[i].y);
                it != sweep.end(); sweep.erase(it++)) {
        int j = it->second;
        P d = ps[i] - ps[j];
        if (d.y > d.x) break;
        edges.push_back({d.y + d.x, i, j});
      }
      sweep[-ps[i].y] = i;
    }
    for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p.y);
  }
  return edges;
}
```

kdTree.h

Description: KD-tree (any dimension)

1f90e6, 73 lines

```
using T = long long;
constexpr int DIM = 2;
using P = array<T, DIM>;
const T INF = numeric_limits<T>::max();

struct Node {
  P pt; // if this is a leaf, the single point in it
  P lo_bound, hi_bound;
  Node *first = 0, *second = 0;

  T distance(const P& p) { // min squared distance to a point
    T r = 0;
    rep(i, 0, DIM) {
      T d = p[i] - max(lo_bound[i], min(hi_bound[i], p[i]));
      r += d * d;
    }
    return r;
  }

  Node(vector<P>&& vp) : pt(vp[0]) {
```

```
    rep(i, 0, DIM) {
      lo_bound[i] = INF; hi_bound[i] = -INF;
    }
    for (const P & p : vp) {
      rep(i, 0, DIM) {
        lo_bound[i] = min(lo_bound[i], p[i]);
        hi_bound[i] = max(hi_bound[i], p[i]);
      }
    }
    if (sz(vp) > 1) {
      // split on x if width >= height (not ideal...)
      pair<T, int> biggest = { -1, -1};
      rep(i, 0, DIM)
      biggest = max(biggest, {hi_bound[i] - lo_bound[i], i});
      int i = biggest.second;
      sort(all(vp), [&](const P & a, const P & b) { return a[i]
           < b[i]; });
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = sz(vp) / 2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node( {all(vp)})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return {node->distance(p), node->pt};
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ... }, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" eefdf5, 88 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  Q rot, o; P p = arb; bool mark;
  P& F() { return r()->p; }
  Q& r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}
Q makeEdge(P orig, P dest) {
  Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
  H = r->o; r->r()->r() = r;
  rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
  r->p = orig; r->F() = dest;
  return r;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e->o = H; H = e; e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
  return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
  q.push_back(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  return pts;
}
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}(n^2)$

"Point3D.h" 5b45fc, 49 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

Strings (9)

Regex.h

Description: You can use the following special characters: `^$.*?|()[]{}`
Time: $\mathcal{O}(NM)$

<regex>, <iostream> 88154e, 15 lines

```
using namespace std;

int main () {
  string s ("this subject has a submarine as a subsequence");
  smatch m; regex e ("(sub)([^ ]*)");   // matches words
       beginning by "sub"
  // regex search will match the first occurence
  if(regex_match(begin(s), end(s), regex("this .+"))) cout << "
       OK" << endl;
  while (regex_search (s, m, e)) {
    for (auto x : m) cout << x << " ";
    cout << endl; s = m.suffix().str();
  }
  s = "this subject has a submarine as a subsequence";
  cout << regex_replace (s,e,"sub-$2");
  return 0;
}
```

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.
Time: $\mathcal{O}(n)$

344602, 27 lines

```
vi pi(const string& s) {
  vi p(sz(s));
  rep(i,1,sz(s)) {
    int g = p[i-1];
    while (g && s[i] != s[g]) g = p[g-1];
    p[i] = g + (s[i] == s[g]);
  }
  return p;
}

vi match(const string& s, const string& pat) {
  vi p = pi(pat + '\0' + s), res;
  rep(i,sz(p)-sz(s),sz(p))
    if (p[i] == sz(pat)) res.push_back(i - 2 * sz(pat));
  return res;
}

vvi automaton(string s, char ch, int charSize) {
  s += '$'; vi ff = pi(s);
  vvi aut(sz(s), vi(charSize));
  rep(i, 0, sz(s)) {
    rep(c, 0, charSize) {
      aut[i][c] = ((i > 0 && ch + c != s[i]) ? aut[ff[i - 1]][c
          ] : (i + (ch + c == s[i])));
    }
  }
  return aut;
}
```

Zfunc.h

Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$

ee09e2, 12 lines

```
vi Z(const string& S) {
  vi z(sz(S));
  int l = -1, r = -1;
  rep(i,1,sz(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  }
  return z;
}
```

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).
Time: $\mathcal{O}(N)$

e7ad79, 13 lines

```
array<vi, 2> manacher(const string& s) {
  int n = sz(s);
  array<vi,2> p = {vi(n+1), vi(n)};
  rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
  }
  return p;
}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(v.begin(), v.begin()+minRotation(v), v.end());`
Time: $\mathcal{O}(N)$

d07a42, 8 lines

```
int minRotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
    if (s[a+k] > s[b+k]) { a = b; break; }
  }
  return a;
}
```

SuffixArray.h

Description: Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i] = lcp(sa[i], sa[i-1])`, `lcp[0] = 0`. The input string must not contain any zero bytes.
Time: $\mathcal{O}(n \log n)$

38db9f, 23 lines

```
struct SuffixArray {
  vi sa, lcp;
  SuffixArray(string& s, int lim=256) { // or basic_string<int>
    int n = sz(s) + 1, k = 0, a, b;
    vi x(all(s)+1), y(n), ws(max(n, lim)), rank(n);
    sa = lcp = y, iota(all(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
      p = j, iota(all(y), n - j);
      rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
      fill(all(ws), 0);
      rep(i,0,n) ws[x[i]]++;
      rep(i,1,lim) ws[i] += ws[i - 1];
      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
      swap(x, y), p = 1, x[sa[0]] = 0;
      rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
        (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
    }
    rep(i,1,n) rank[sa[i]] = i;
```

```
    for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
      for (k && k--, j = sa[rank[i] - 1];
          s[i + k] == s[j + k]; k++);
  }
};
```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).
Time: $\mathcal{O}(26N)$

aae0b8, 50 lines

```
struct SuffixTree {
  enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
  int toi(char c) { return c - 'a'; }
  string a; // v = cur node, q = cur position
  int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;

  void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
      if (t[v][c]==-1) { t[v][c]=m;  l[m]=i;
        p[m++]=v; v=s[v]; q=r[v];  goto suff; }
      v=t[v][c]; q=l[v];
    }
    if (q==-1 || c==toi(a[q])) q++; else {
      l[m+1]=i;  p[m+1]=m;  l[m]=l[v];  r[m]=q;
      p[m]=p[v];  t[m][c]=m+1;  t[m][toi(a[q])]=v;
      l[v]=q;  p[v]=m;  t[p[m]][toi(a[l[m]])]=m;
      v=s[p[m]];  q=l[m];
      while (q<r[m]) { v=t[v][toi(a[q])];  q+=r[v]-l[v]; }
      if (q==r[m])  s[m]=v;  else s[m]=m+2;
      q=r[v]-(q-r[m]);  m+=2;  goto suff;
    }
  }

  SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
  }

  // example: find longest common substring (uses ALPHA = 28)
  pii best;
  int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
      mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
      best = max(best, {len, r[node] - len});
    return mask;
  }
  static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
  }
};
```

Aho3S.h

Description: Aho-Corasick by Kak Ucup

08beab, 51 lines

```
struct AhoCorasick {
  int trie[N][26], fail[N], saiz;
  AhoCorasick() {
    memset(trie[0],-1,sizeof trie[0]);
    saiz = 0;
  }
  void add(string str) {
    int cur = 0;
    for(int i = 0 ; i < str.length() ; i++) {
      //checkChar(str[i]);
      int nex = str[i] - 'a';
      if(trie[cur][nex] == -1) {
        trie[cur][nex] = ++saiz;
        memset(trie[saiz],-1,sizeof trie[saiz]);
      }
      cur = trie[cur][nex];
    }
  }
  void build() {
    queue<int> q;
    fail[0] = 0;
    for(int i = 0 ; i < 26 ; i++)
      if(trie[0][i] == -1)
        trie[0][i] = 0;
      else {
        int nex = trie[0][i];
        fail[nex] = 0;
        q.push(nex);
      }
    while(!q.empty()) {
      int now = q.front();
      q.pop();
      for(int i = 0 ; i < 26 ; i++)
        if(trie[now][i] == -1)
          trie[now][i] = trie[fail[now]][i];
        else {
          int nex = trie[now][i];
          fail[nex] = trie[fail[now]][i];
          q.push(nex);
        }
    }
  }
  int getIndex(string str) {
    int cur = 0;
    for(int i = 0 ; i < str.length() ; i++) {
      //checkChar(str[i]);
      cur = trie[cur][str[i] - 'a'];
    }
    return cur;
  }
};
```

Various (10)

10.1 Dynamic programming

Alien.h

Description: Solution for alien
Time: $\mathcal{O}(NlogK)$

8382fb, 39 lines

```
pi trial(lint l){
  cht.clear();
  for(int i=1; i<=v.size(); i++){
    cht.add_line(2 * 2 * v[i-1].first, dp[i-1].first +
      2ll * v[i-1].first * v[i-1].first, dp[i-1].second);
    dp[i] = cht.query(-v[i-1].second);
    dp[i].first += 2ll * v[i-1].second * v[i-1].second + l; //
        l is penalty
    dp[i].second++;
    if(i != v.size()){
      lint c = max(0ll, v[i-1].second - v[i].first);
      dp[i].first -= 2 * c * c;
    }
  }
  return dp[v.size()];
}

long long take_photos(int n, int m, int k, std::vector<int> r,
     std::vector<int> c) {
  vector<pi> w;
  for(int i=0; i<n; i++){
    if(r[i] > c[i]) swap(r[i], c[i]);
    w.push_back({r[i]-1, c[i]});
  }
  sort(w.begin(), w.end(), [&](const pi &a, const pi &b){
    return pi(a.first, -a.second) < pi(b.first, -b.second);
  });
  for(auto &i : w){
    if(v.empty() || v.back().second < i.second){
      v.push_back(i);
    }
  }
  lint s = 0, e = 2e12;
  while(s != e){
    lint m = (s+e)/2;
    // See how many groups are made with penalty 2*m+1
    if(trial(2 * m + 1).second <= k) e = m;
    else s = m+1;
  }
  return trial(s * 2).first / 2 - s * k;
}
```

SlopeTrick.h

Description: Slope Trick making it increasing with cost to up and down different
Time: $\mathcal{O}(N \log N)$

453f59, 41 lines

```
int solve() {
  int n; cin >> n;
  vector<LL> isi(n);
  vector<PLL> cost(n);
  trav(cur, isi) cin >> cur;
  trav(cur, cost) cin >> cur.fi;
  trav(cur, cost) cin >> cur.se;
  LL ans = 0;
  priority_queue <PLL> solve;
  rep(i, 0, n) {
    auto &cur = cost[i];
    // Push ke isi[i], gradiennya cur.fi + cur.se
    solve.push({isi[i], cur.fi + cur.se});
    // Push lagi buat kurangin gradien
    solve.push({max(isi[i], solve.top().fi), -cur.fi});
    ans += (solve.top().fi - isi[i]) * cur.fi;
    // Maintain minimum
    while (solve.size() >= 2){
      PLL now = solve.top(); solve.pop();
      PLL pre = solve.top(); solve.pop();
      // Merge dua slope
      if(now.fi == pre.fi){
        now.se += pre.se;
        solve.push(now);
        continue;
      }else if(pre.fi < now.fi && now.se <= 0){
        // Slope trick, geser opt
        pre.se += now.se;
```

```
        solve.push(pre);
        ans += (now.fi - pre.fi) * now.se;
        continue;
      }else{
        solve.push(pre);
        solve.push(now);
        break;
      }
    }
  }
  cout << ans << endl;
  return 0;
}
```

SosDP.h

Description: Consider using FWHT

481c5f, 9 lines

```
void sos() {
  //Dp[mask] contain all numbers of submask
  for (int i = 0; i <= 23; i++)
    for (int mask = (1LL << 24) - 1; mask >= 0; mask--)
      if (mask & (1LL << i)) dp[mask] += dp[mask ^ (1LL << i)];
  LL ans = 0;
  for (int mask = (1LL << 24) - 1; mask >= 0; mask--)
    dp[mask] = n - dp[mask], ans ^= (dp[mask] * dp[mask]);
}
```

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \leq f(a,d)$ and $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.

Time: $\mathcal{O}\left(N^2\right)$

701e4d, 20 lines

```
int main() {
  for (int i = 1; i < n; i++) {
    memo[i][i + 1] = isi[i] + isi[i + 1];
    opt[i][i + 1] = i;
  }
  for (int i = 2; i <= n; i++) {
    //Compute for i+1 segment
    for (int j = 1; j + i <= n; j++) {
      LL cur = LINF;
      for (int k = opt[j][j + i - 1]; k <= opt[j + 1][j + i]; k
          ++) {
        LL now = memo[j][k] + memo[k + 1][j + i];
        if (cur > now) {
          cur = now;
          opt[j][j + i] = k;
        }
      }
      memo[j][j + i] = cur + (psum[j + i] - psum[j - 1]);
    }
  }
}
```

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i) \leq k < hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.

Time: $\mathcal{O}\left((N + (hi - lo))\log N\right)$

d38d2b, 18 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k, v); }
  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

10.2 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

10.3 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

10.3.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

FastMod.h

Description: Compute $a\%b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to a (mod b) in the range $[0, 2b)$.

751a02, 8 lines

```
typedef unsigned long long ull;
struct FastMod {
  ull b, m;
  FastMod(ull b) : b(b), m(-1ULL / b) {}
  ull reduce(ull a) { // a % b + (0 or b)
    return a - (ull)((__uint128_t(m) * a) >> 64) * b;
  }
};
```

Random.h

Description: Random using mersenne twister

Usage: `rng()`

<random>, <chrono> 99f56c, 7 lines

```
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().
    count());
// shuffle(isi.begin(),isi.end(),rng);
LL getRange(LL a, LL b){
   LL ran = b-a+1;
   return (rng()%ran)+a;
}
```

ClockTime.h

Description: Elapsed time from the beginning of running program

Usage: `cek_time()`

0453a9, 5 lines

```
clock_t first_attempt = clock();
inline void cek_time(){
  clock_t cur = clock()- first_attempt;
  cerr<<"TIME : "<<(double) cur/CLOCKS_PER_SEC<<endl;
}
```

FastInput.h

Description: Read an integer from stdin. Usage requires your program to pipe in input from file.

Usage: `./a.out < input.txt`

Time: About 5x as fast as cin/scanf.

7b3c70, 17 lines

```
inline char gc() { // like getchar()
  static char buf[1 << 16];
  static size_t bc, be;
  if (bc >= be) {
    buf[0] = 0, bc = 0;
    be = fread(buf, 1, sizeof(buf), stdin);
  }
  return buf[bc++]; // returns 0 on EOF
}

int readInt() {
  int a, c;
  while ((a = gc()) < 40);
  if (a == '-') return -readInt();
  while ((c = gc()) >= 48) a = a * 10 + c - 480;
  return a - 48;
}
```

BumpAllocator.h

Description: When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation.

745db2, 8 lines

```
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
  static size_t i = sizeof buf;
  assert(s < i);
  return (void*)&buf[i -= s];
}
void operator delete(void*) {}
```

10.4 Known Problems

StableMarriage.h

Description: While there is a free man m: let w be the most preferred woman to whom he has not yet proposed, and propose m to w. If w is free, or is engaged to someone whom she prefers less than m, match m with w, else deny proposal.

MoserCircle.h

Description: Determine the number of pieces into which a circle is divided if n points on its circumference are joined by chords with no three internally concurrent. Solution: g(n) = nC4 + nC2 + 1.

ChickenMcNugget.h

Description: Chicken McNugget Theorem states that for any two relatively prime positive integers m,n, the greatest integer that cannot be written in the form am+bn for nonnegative integers a,b is mn − m - n.

EulerFaceFormula.h

Description: V − E + F = 2 [V: vertices E: edges F: faces]

CayleyFormula.h

Description: There are n^{n-2} spanning trees of a complete graph with n labeled vertices. Spanning Tree of Complete Bipartite Graph is $N^{M-1} * M^{N-1}$.

PickTheorem.h

Description: Pick's Theorem: A = i + b/2 − 1. A is Area, I is internal points, and B is Border points .

JosephusProblem.h

Description: There are n person in a table waiting to be executed. Person 1 hold a knife. Each step whoever has the knife, kill the person next to him. Who's alive at the end?

f8e6ed, 12 lines

```
int x = 0;
for (int i = 2; i <= n; ++i)
  x = (x + i) % i;

int josephus(int n, int k) {
    if (n == 1) return 0;
    if (k == 1) return n-1;
    if (k > n) return (josephus(n-1, k) + k) % n;
    int cnt = n / k, res = josephus(n - cnt, k) - (n % k);
    res += (res < 0 ? n : (res / (k - 1)));
    return res;
}
```

ErdosGallai.h

Description: Given degree of n nodes. Is it possible to build the graph?

f00885, 20 lines

```
sort(d+1, d+n+1, greater<int>);
for (i=1;i<=n;i++)
    x[i] = x[i-1] + d[i];
if (x[n]&1) {
    printf("Not possible\n");
    continue;
}
can = true;
for (k=1;k<=n;k++) {
    sum = x[k];
    tmp = k*(k-1);
    for (i=k+1;i<=n;i++)
        tmp += min(d[i], k);
    if (sum > tmp) {
        can = false;
        break;
    }
}
if (can) printf("Possible\n");
else printf("Not possible\n");
```

10.5 Minimum-Cut Problems

OpenPit.h

Description: Open Pit

4e4011, 13 lines

```
for (int i = 1; i <= n; i++) {
  int a, b; cin >> a >> b;
  isi[i] = a - b; int m; cin >> m;
  while (m--) { int v; cin >> v; edges.pb({i, v}); }
}
PushRelabel solve(n + 2);
for (int i = 1; i <= n; i++) {
  int curcost = abs(isi[i]);
  if (isi[i] == curcost) ans += curcost, solve.add_edge(0, i,
      curcost);
  else solve.add_edge(i, n + 1, curcost);
}
trav(edge, edges) solve.add_edge(edge.se, edge.fi, INF);
cout << ans - solve.maxflow(0, n + 1) << endl;
```

../graph/2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||b)\&\&(!a||c)\&\&(d||!b)\&\&\ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

Usage:

```
TwoSat ts(number of boolean variables);
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.setValue(2); // Var 2 is true
ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars
```

Time: $\mathcal{O}(N + E)$, where N is the number of boolean variables, and E is the number of clauses.

5f9706, 50 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false, 1 = true
  TwoSat(int n = 0) : N(n), gr(2*n) {}
  int addVar() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }
  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f].push_back(j^1);
    gr[j].push_back(f^1);
  }
  void setValue(int x) { either(x, x); }
  void atMostOne(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i,2,sz(li)) {
      int next = addVar();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }
  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    for(int e : gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = low;
      if (values[x>>1] == -1)
        values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;
  }
  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i,0,2*N) if (!comp[i]) dfs(i);
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
};
```

../data-structures/Sparse2D.h

Description: Offline solve 2d commutative query This query is exclusive 1-based, change to [st, ed). Reduce to 0-based indexing!

Time: $\mathcal{O}\left(N \log^2 N\right)$, query in $\mathcal{O}(1)$

447412, 18 lines

```
const int LOGN = 13;
int sparset[LOGN][LOGN][205][205];
rep(i,0,n) rep(j,0,m) cin >> sparset[0][0][i][j];
rep(i,0,n) rep(logj,1,LOGN) rep(j,0,m-(1<<logj)+1)
  sparset[0][logj][i][j] = min(sparset[0][logj-1][i][j],
    sparset[0][logj-1][i][j+(1<<(logj-1))]);
rep(logi, 1, LOGN) rep(logj, 0, LOGN)
  rep(i,0,n-(1<<logi)+1)
  rep(j,0,m-(1<<logj)+1)
    sparset[logi][logj][i][j] = min(sparset[logi-1][logj][i][j
        ],
      sparset[logi-1][logj][i+(1<<(logi-1))][j]);
cin >> st.fi >> st.se >> ed.fi >> ed.se; st.fi--; st.se--;
int lenrow = 31-__builtin_clz(ed.fi-st.fi);
int lencol = 31-__builtin_clz(ed.se-st.se);
res1 = min(min(min(sparset[lenrow][lencol][st.fi][st.se],
sparset[lenrow][lencol][st.fi][ed.se-(1<<lencol)]),
sparset[lenrow][lencol][ed.fi-(1<<lenrow)][st.se]),
sparset[lenrow][lencol][ed.fi-(1<<lenrow)][ed.se-(1<<lencol)]);
```

NarrowRectangle.h

Description: Define dp[i][x]: the minimum cost to move the first i rectangles such that the last (the i-th) rectangle's leftmost coordinate is x. This will lead to a solution for partial score.

8195b9, 25 lines

```
priority_queue<LL> kiri;
priority_queue<LL, vector<LL>, greater<LL> > kanan;
int main() {
  ios_base::sync_with_stdio(false); cin.tie(0); cout.tie(0);
  cin >> n;
  for (int i = 1; i <= n; i++) cin >> l[i] >> r[i];
  LL ans = 0LL, lzl = 0LL, lzr = 0LL;
  kiri.push(l[1]); kanan.push(l[1]);
  for (int i = 2; i <= n; i++) {
    lzr += r[i - 1] - l[i - 1];
    lzl -= r[i] - l[i];
    LL tmpl = kiri.top() + lzl, tmpr = kanan.top() + lzr;
    if (l[i] <= tmpl) {
      kiri.pop(); kiri.push(l[i] - lzl); kiri.push(l[i] - lzl);
      kanan.push(tmpl - lzr); ans += llabs(tmpl - l[i]);
    } else if (l[i] >= tmpr) {
      kanan.pop(); kanan.push(l[i] - lzr); kanan.push(l[i] -
          lzr);
      kiri.push(tmpr - lzl); ans += llabs(l[i] - tmpr);
    } else {
      kiri.push(l[i] - lzl); kanan.push(l[i] - lzr);
    }
  }
  cout << ans << '\n';
  return 0;
}
```